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Forecasting and reliability

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Abstract

No precise law governs the design of electrical power systems, and due to the errors inherent in the results obtained, confidence in forecasting methods is low. In reality, a large number of possible solutions may arise, and one aims to determine the limits within which they will be found — more precisely, within what boundaries a certain proportion of them lies.

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1 Introduction

1.1 Probability

No precise law governs the design of electrical power systems, and due to the errors inherent in the results obtained, confidence in forecasting methods is low. In reality, a large number of possible solutions may arise, and one aims to determine the limits within which they will be found — more precisely, within what boundaries a certain proportion of them lies.

Nothing can be predicted through a purely deductive process; it is recognized by the axioms of any logic that no deductive process can lead to the demonstration of a proportion related to forecasting.

No two events are identical, yet certain factors common to them must be selected and categorized as favorable or unfavorable.

The probability of success in an experiment is defined as the ratio between the number of possible cases in which a favorable result occurs and the total number of possible outcomes. Ultimately, nothing can occur twice in exactly the same way; this introduces the ratio of two infinite numbers. However, in practice, possibilities are grouped discretely, and the ratio is taken between the number of possible results in the favorable subgroup and the total number of possibilities. This statement cannot be considered a vicious circle.

Starting from this point, the mathematical procedure follows the path of measurement theory, defining an event as an element belonging to a set; however, this consideration is avoided here in order to ensure simplicity and speed.

In light of the above ideas, Bernoulli's theorem becomes a tautology. It can thus be stated as follows: if the ratio between the number of favorable results and the number of possible results in a single trial is p, then the ratio between the number of ways in which the above condition occurs in n trials and the total number of possible results tends toward 1 as n tends toward infinity. A careful examination of this theorem shows that, in reality, what has been stated indeed occurs, and the statement is self-evident or tautological.

1.2 Practical Implication of Definition of Probability

The probability of an event is, in fact, an important and the only scientific basis we have for a rational decision. In practical terms, the ratio of the number of ways—or the probability—represents the degree of confidence we have in a rational assumption or statement we hold, and it is the basis upon which we could rationally wager.

Nevertheless, one must respond to a criticism, since there exists a new logical sequence in the reasoning above. If it can be said that the probability of an event corresponds to the chance at which a reasonable person would be equally willing to propose and accept wagers on a trial, then we would have a method for putting the theory into practice [1]. However, there are no serious reasons to affirm this. There are many people who accept Macpherson's law. It must be noted that such convictions can remain fixed in a person's memory and may obscure many cases in which everything went well. It appears that Macpherson considers himself a conventional inductionist and may make the same claims as any other inductionist.

Probability means only the ratio between favorable cases and the total number of possible cases, and this definition says nothing about the events that have actually occurred. In fact, Macpherson could claim—and indeed does claim—that, from a logical standpoint, he is "probably" right.

Nevertheless, there is a factor that lends support to the reasoning of classical inductionists, at least in engineering practice. Macpherson's law leads to impractical results. Let us suppose, for example, that we have concluded that 90% of the total load values that a system was required to sustain a few years ago fall between 100 and 120 MW [2]. According to Macpherson, we should certainly expect values below or above these limits. But how much above or below? And which limits should be considered? What is preferable—to risk failing to supply power, or to oversize the generation capacity in an irrational manner?

2 Determining Probability from Frequency data

In practice, we rarely possess data that allow for a direct evaluation of the probability that an event will occur. This probability must therefore be assessed based on frequency data. For example, suppose that in ten draws, one black ball is obtained. Let us then consider the probability p_2, such that the probability of the outcomes is p_1. Naturally, we would wish to choose p_1 so that p_2 is maximized.

Thus, p_2 will be equal to the ratio between the number of ways in which one can draw ten balls (one of which is black) and the number of ways in which one can draw ten balls, assuming that everything indicates the proportion of black balls is p_1. It is evident that in our case p_2 is maximal when $p_1 = 1/10$.

This is the principle of maximum likelihood, based on which we select hypotheses that lead to the most probable observation.

Having clarified the nature of the concept of probability, it now remains to show how the principle of maximum likelihood provides a method for obtaining a law on the basis of an adequate set of data, and how an appropriate form of that law can be determined.

2.1 Regression and Method of Least Squares

Let us consider a random variable Y_i, assumed to have a normal distribution with a mean μ, a variance δ^2, and (legitimately, in this case) let us suppose that $Y_i \sim N(\mu, \delta^2)$.

Furthermore, let us assume that the relationship is homoscedastic, meaning that its variance is independent of other variables (that is, δ^2 is independent of Y).

Given a set of samples (y_i, x_i), an estimate of μ and β can be obtained as indicated below.

The probability that

$$x \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{1}{2\sigma^2}[y_i - \mu - \beta(x_i - x)]^2\right\} \delta y_i \tag{1}$$

The likelihood L

$$L = \prod_{i=1}^{n} \left\{\frac{1}{\sqrt{2\pi\sigma^2}} \exp -\frac{1}{2\sigma^2}[y_i - \mu - \beta(x_i - x)]^2\right\} \tag{2}$$

Assuming

$$\sum_1^n [y_i - \mu - \beta(x_i - x)]^2 \tag{3}$$

Thus, a law of adjustment for a sequence of data is obtained, and we are provided with a rational method for refining or fitting even the law itself [3].

2.2 Choosung an appropriate law

The postulate of simplicity, as defined by Jeffreys, is demonstrated in [23]. The demonstration provided does not show why only N_0 possible laws exist (although this has been omitted as something self-evident), nor why the highest probability is associated with the simplest laws. These points are discussed below.

Moreover, Jeffreys seems to exclude differential equations from the class of laws; however, since $N_0 + N_0 = N_0$, any finite number of other applicable schemes may exist, and in practice, one of this type is used in what follows.

We consider that the best proof of this postulate is experience itself.

We shall demonstrate that, at any level of knowledge, among two or more possible laws derived from the available data that relate a certain number of factors, the simplest law is the most probable.

First, let us determine how many laws are possible. The apparent answer is that their number is infinite. In other words, the possibility that a law might be represented by a high-order polynomial or a higher-order differential equation cannot be entirely eliminated; nevertheless, no matter how improbable a law may be, it will always have a finite probability, however small.

We now face the following problem: there appears to be an infinite number of laws, each with a finite probability. This infinity of laws covers all possible probabilities — that is, the sum of all their probabilities equals unity (certainty). We then ask: which infinite series of finite numbers can have a finite sum?

A certain answer is given by the terms of a convergent infinite series, for example:

$$1 = \frac{1}{2} + \frac{1}{2} + \frac{1}{8} + \frac{1}{16} \dots \quad (4)$$

The number of terms in such infinite series is N_0, the first and smallest **transfinite number**. Therefore, it is easy to show that there exist only N_0 possible laws and that all these laws have **rational probabilities**. The proof is carried out as follows.

The **sum of the probabilities** of all possible laws is finite (equal to unity). Consequently, the number of laws with any given probability $P$is less than $1/P$ and is finite if the law is possible (that is, if it has a finite probability). Likewise, the number of laws with a probability greater than P is also finite.

Arrange the probabilities in a **progression** [4] as follows: place first the greatest probability, and then arrange the rest so that they form a decreasing progression. Each element in this progression is a finite number, and therefore the progression is uniquely determined. Thus, the probabilities can be assigned a **rank** — 1, 2, 3, and so on — and their total number may be finite or N_0, that is, the number of all integers.

From the consideration of the transformation

$$y = \frac{Z}{1-Z^2} \quad (5)$$

it is clear that the above demonstration does not depend on whether we take the probability of certainty as 1 or as infinity.

Hence, we have shown that the probabilities of all possible laws can be arranged as an **ordered and convergent series**.

It remains to show why the **highest probability** in this series belongs to the **simplest law**. At a given level of our knowledge, we have at our disposal a finite number of data. This allows for the determination of a finite number of coefficients in a law. For example, two pairs of readings allow us to draw a straight line, while three readings make it possible to draw a quadratic curve or to verify a straight line.

Thus, assuming that we have learned from experience, our knowledge is such that at an initial stage the probability of a **simple law** is greater than that of a more complex one.

Let us now consider successive sets of verifying data. This does not affect the prior probability of a more complex law, $P(q/h) = 1$; that is, the initial quantities of data are not sufficient to determine the coefficients of more complex laws, and thus the probability of these data under the proposed variability of the law is certainly equal.

Therefore, if we exclude the hypothesis that the highest probabilities are associated with the simplest laws, the **posterior probabilities** of a more complex law would remain greater than those of the simpler ones (although the most complex law might remain unverified), when we have a finite number of data — since the sum of the series of probabilities must equal unity. But this conclusion contradicts the result of the previous paragraph.

These findings can also be interpreted as follows: the **series of complexity** must correspond **biuniquely** to the first term, since $N_0 - n = N_0$.

This demonstrates that the probabilities of possible laws that can be used for forecasting form a **convergent series**, with the **highest probability** being associated with the **simplest law**.

3 Conclusions

It can be shown that the above demonstration of the **postulate of simplicity** is subject to **Kronecker's critique**, which—if carried to its ultimate consequences—could undermine all of analysis, including infinitesimal calculus. Nevertheless, the criticism that can be raised here is that any rational probability can be associated with any law, and that the entire problem lies merely in reducing the number of possible laws from $N_0$to a number greater than a given one. Therefore, for **practical purposes**, such criticism is of no significance.

Moreover, the conclusions are consistent with the results of **regression analysis**: the more complex a law is, the greater the **dispersion of its regression coefficients**. Hence, it is found that the more complex the law, the **wider the forecast limits**. And precisely in order to obtain results that are as accurate as possible, we seek to choose **the simplest possible laws**.

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