

Sensorless Vector Control of a Surface Permanent Magnet Synchronous Motor

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Abstract

This paper analyzes the operation of the surface permanent magnet synchronous motor (SPMSM) in a sensorless configuration, with the main objective of speed estimation. It is presented a vector control strategy independent of position sensors, implemented through a voltage inverter with prescribed currents. Advanced algorithms, including the Gopinath observer, are used to accurately determine the rotor speed and position. This plays an essential role in increasing the stability and accuracy of the control system, by reducing estimation errors and ensuring a rapid response to load variations. Performance validation was performed through simulations in the MATLAB/Simulink environment, with the results demonstrating both the accuracy and reliability of sensorless control under various operating conditions. The conclusions highlight the efficiency of the proposed method and its relevance for the development of modern electric drive technologies.

Keywords: surface permanent magnet synchronous motor, vector control, Gopinath observer.

1 Introduction

Surface permanent magnet synchronous motor (SPMSM) has become benchmark solutions in modern applications due to their low energy consumption, advantageous power-to-volume ratio, and ability to respond quickly to load variations. They are frequently found in traction systems for electric vehicles, in automated production lines with industrial robots, but also in other drives that require high performance. For the proper operation of these machines, very precise control of the rotor position and speed is required [1]–[4]. In the conventional approach, this involves integrating encoder or resolver sensors, but these increase the price of the equipment, complicate assembly and can become vulnerable points in demanding industrial conditions.

An alternative to mechanical sensors is estimation techniques based on measured electrical signals. Multiple methods are described in the specialized literature that uses both mathematical models of the engine and numerical processing algorithms. For example, by applying the equations of state and using stator currents and voltages, information about the rotor position and speed can be deduced. Another method relies on magnetic flux analysis to determine the stator space vector, also based on stator voltages and currents [2], [5]–[8].

In this paper, an SPMSM is studied in a sensorless control regime, with the emphasis on determining the rotor speed using the reduced order adaptive observer developed by Gopinath. The theoretical foundations of the motor and the vector control strategies applicable to sensorless control are described. A control scheme without position sensors is also presented, the motor being powered by a voltage inverter with prescribed currents. [5], [8]–[11]. The performance of the method was evaluated through simulations performed in MATLAB/Simulink, the results confirming the accuracy and stability of the solution.

2 Mathematical model

2.1 Permanent magnet synchronous motor model

To deduce the structure of the Gopinath reduced order adaptive observer, we start from the SPMSM engine model represented in a d-q axis rotor mobile system. Thus, the stator voltage equations are,

$$u_d = R_s i_d - p\omega_r L_s i_q + L_s \frac{di_d}{dt} \quad (1)$$

$$u_q = R_s i_q + p\omega_r L_s i_d + p\omega_r \phi_0 + L_s \frac{di_q}{dt} \quad (2)$$

Where,

$$i_s = i_d + ji_q, u_s = u_d + ju_q \quad (3)$$

The calculation relationship for the electromagnetic torque of the surface permanent magnet synchronous machine is

$$T = \frac{3}{2} p \phi_0 i_q \quad (4)$$

To determine the speed of the car, the dynamic equation of motion is integrated,

$$J \frac{d\omega_r}{dt} = T - T_l - F\omega_r \quad (5)$$

Thus, the canonical form of the system of equations of the surface permanent magnet synchronous machine model results,

$$\begin{aligned} \frac{di_d}{dt} &= -\frac{R_s}{L_s} i_d + p\omega_r i_q + \frac{1}{L_s} u_d \\ \frac{di_q}{dt} &= -\frac{R_s}{L_s} i_q - p\omega_r \frac{\phi_0}{L_s} - p\omega_r i_d + \frac{1}{L_s} u_q \\ \frac{d}{dt} \omega_r &= \frac{3}{2J} p \phi_0 i_q - \frac{F}{J} \omega_r - \frac{T_l}{J} \\ \frac{d}{dt} \theta_r &= \omega_r \end{aligned} \quad (6)$$

Where,

R_s - the stator resistance

L_s - the stator inductance

ϕ_0 - the permanent magnet flux linkage

p - the pole pair number

ω_r - the rotor speed
 θ_r - the angular position
 F - the friction coefficient
 T_l - the load torque
 J - the moment inertia

Studying equations (6), it is observed that the multiplication of two state variables ($\omega_r i_d$) and ($\omega_r i_q$) is performed, which results in a non-linear system. This shortcoming must be corrected in order to implement the Gopinath reduced order adaptive observer, thus transforming the system into a linear one. Thus, the system of equations (6) becomes linear if the order of the state variables is changed, resulting in,

$$\frac{d}{dt} \begin{bmatrix} \omega_r \\ \theta_r \\ i_d \\ i_q \end{bmatrix} = \begin{bmatrix} -\frac{F}{J} & 0 & 0 & \frac{3}{2J} p \phi_0 \\ 1 & 0 & 0 & 0 \\ p i_q & 0 & -\frac{R_s}{L_s} & 0 \\ -\frac{p}{L_s} (\phi_0 + L_s i_d) & 0 & 1 & -\frac{R_s}{L_s} \end{bmatrix} \begin{bmatrix} \omega_r \\ \theta_r \\ i_d \\ i_q \end{bmatrix} + \begin{bmatrix} 0 & 0 & \frac{1}{J} \\ 0 & 0 & 0 \\ \frac{1}{L_s} & 0 & 0 \\ 0 & \frac{1}{L_s} & 0 \end{bmatrix} \begin{bmatrix} u_d \\ u_q \\ T_l \end{bmatrix} \quad (7)$$

Where,

$$A = \begin{bmatrix} -\frac{F}{J} & 0 & 0 & \frac{3}{2J} p \phi_0 \\ 1 & 0 & 0 & 0 \\ p i_q & 0 & -\frac{R_s}{L_s} & 0 \\ -\frac{p}{L_s} (\phi_0 + L_s i_d) & 0 & 1 & -\frac{R_s}{L_s} \end{bmatrix} \quad (8)$$

$$B = \begin{bmatrix} 0 & 0 & \frac{1}{J} \\ 0 & 0 & 0 \\ \frac{1}{L_s} & 0 & 0 \\ 0 & \frac{1}{L_s} & 0 \end{bmatrix} \quad (9)$$

2.2 Gopinath observer model

The Gopinath observer is designed to estimate electrical and mechanical quantities in systems where sensors have been eliminated. In this paper, the motor speed and the rotor position relative to the stator will be estimated. These estimates are possible if we intervene on the state vectors,

$$x = \begin{bmatrix} \omega_r \\ \theta_r \\ i_d \\ i_q \end{bmatrix} = \begin{bmatrix} x_e \\ y \end{bmatrix} \quad (10)$$

Where,

$$x_e = \begin{bmatrix} \omega_r \\ \theta_r \end{bmatrix}, y = \begin{bmatrix} i_d \\ i_q \end{bmatrix} \quad (11)$$

With these adjustments, the matrix equation (7) becomes,

$$\frac{d}{dt} \begin{bmatrix} x_e \\ y \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \cdot \begin{bmatrix} x_e \\ y \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \cdot \begin{bmatrix} u_d \\ u_q \\ T_l \end{bmatrix} \quad (12)$$

Where,

$$a_{11} = \begin{bmatrix} -\frac{F}{J} & 0 \\ 1 & 0 \end{bmatrix}, a_{21} = \begin{bmatrix} p i_q & 0 \\ -\frac{p}{L_s}(\phi_0 + L_s i_d) & 0 \end{bmatrix} \quad (13)$$

$$a_{12} = \begin{bmatrix} 0 & \frac{3}{2J} p \phi_0 \\ 0 & 0 \end{bmatrix}, a_{22} = \begin{bmatrix} -\frac{R_s}{L_s} & 0 \\ 0 & -\frac{R_s}{L_s} \end{bmatrix} \quad (14)$$

$$b_1 = \begin{bmatrix} 0 & 0 & \frac{1}{J} \\ 0 & 0 & 0 \end{bmatrix}, b_2 = \begin{bmatrix} \frac{1}{L_s} & 0 & 0 \\ 0 & \frac{1}{L_s} & 0 \end{bmatrix} \quad (15)$$

In the specialized literature [12],[13] the expression of the Gopinath observer is,

$$\frac{d}{dt} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = F \cdot \begin{bmatrix} \omega_r \\ \theta_r \end{bmatrix} + G \cdot \begin{bmatrix} u_d \\ u_q \\ T_l \end{bmatrix} + H \cdot \begin{bmatrix} i_d \\ i_q \end{bmatrix} \quad (16)$$

Where, $z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$ (17), the vector of state variables of the Gopinath observer.

$$\begin{bmatrix} \omega_r \\ \theta_r \end{bmatrix} = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + L \cdot \begin{bmatrix} i_d \\ i_q \end{bmatrix} \quad (18)$$

$L = \begin{bmatrix} l_{11} & l_{12} \\ l_{21} & l_{22} \end{bmatrix}$ (19), matrix of unknown parameters. This matrix determines the stability of the observer and it is obtained by the pole placement method.

$$F = a_{11} - L \cdot a_{21} \quad (20)$$

$$G = b_1 - L \cdot b_2 \quad (21)$$

$$H = a_{12} - L \cdot a_{22} + F \cdot L \quad (22)$$

The substitutions of expressions (13) and (19) will be made in relation (20) and the matrix expression F will be obtained,

$$F = \begin{bmatrix} -\frac{F}{J} - l_{11} p i_q + l_{12} \frac{p}{L_s}(\phi_0 + L_s i_d) & 0 \\ 1 - l_{21} p i_q + l_{22} \frac{p}{L_s}(\phi_0 + L_s i_d) & 0 \end{bmatrix} \quad (23)$$

The polynomial that determines the poles assigned to the observer is,

$$P_0(s) = \det(s \cdot I_2 - F) = s \left[s + \frac{F}{J} + l_{11} p i_q - l_{12} \frac{p}{L_s}(\phi_0 + L_s i_d) \right] \quad (24)$$

By solving the above equation, a null solution is determined. It is further assumed that there is a solution $p_1 \neq 0$ and it is considered that the constants $l_{11} = l_{21} = l_{22} = 0$ do not influence the dynamics of the process. Thus, with these working hypotheses, the matrices G and H are determined,

$$G = \begin{bmatrix} 0 & -\frac{l_{12}}{J} & -\frac{1}{J} \\ 0 & 0 & 0 \end{bmatrix} \quad (25)$$

$$H = \begin{bmatrix} 0 & \frac{3}{2J}p\phi_0 + l_{12}\left(\frac{R_s}{L_s} - \frac{F}{J} + l_{12}\frac{p}{L_s}(\phi_0 + L_s i_d)\right) \\ 0 & l_{12} \end{bmatrix} \quad (26)$$

After substitutions in equation (16), the detailed expression of the Gopinath observer is obtained,

$$\begin{aligned} \frac{dz_1}{dt} = & \left[-\frac{F}{J} + l_{12}\frac{p}{L_s}(\phi_0 + L_s i_d)\right] z_1 - \frac{l_{12}}{L_s} u_q - \frac{T_l}{J} + \\ & + \left(\frac{3}{2J}p\phi_0 + l_{12}\frac{R_s}{L_s}\right) i_q + l_{12}\left[-\frac{F}{J} + l_{12}\frac{p}{L_s}(\phi_0 + L_s i_d)\right] i_q \end{aligned} \quad (27)$$

$$\frac{dz_2}{dt} = z_1 + l_{12} \cdot i_q \quad (28)$$

If equation (27) is solved and relation (18) is taken into account, the two estimated electrical quantities are determined as follows: speed and angular position of the rotor, $(\omega_{r_estim}, \theta_{r_estim})$,

$$\omega_{r_estim} = z_1 + l_{12} \cdot i_q \quad (29)$$

$$\theta_{r_estim} = z_2 \quad (30)$$

Where,

$$l_{12} = \left(p_1 + \frac{F}{J}\right) \cdot \frac{L_s}{p(\phi_0 + L_s i_d)} \quad (31)$$

3 Motor control with current controlled voltage inverter

The modelling and simulation of the vector control of the SPMSM powered by a current controlled VSI voltage inverter in a sensorless system is presented in the block diagram in the figure Fig.1.

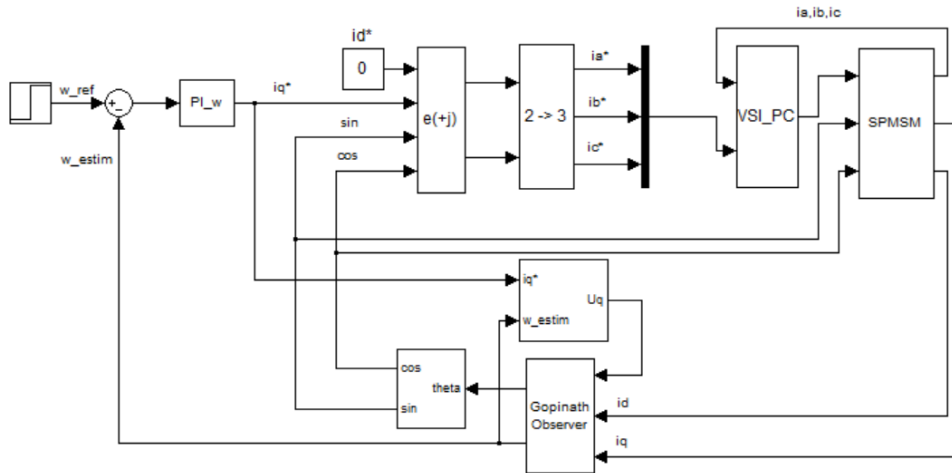


Figure 1. Control block diagram of SPMSM in sensorless system

It is considered as a regulation strategy for the synchronous motor with magnets, imposing the maintenance of zero reactive current ($i_d^* = 0$) and the active component (i_q^*) is the one that controls the electromagnetic torque developed by the motor. This current component is the result of the speed controller „PI-w”. Both current components are considered to be in the rotating system joint with the rotor. Transforming the two components (i_d^*, i_q^*) into the fixed three-phase system, ($e^{+j}, 2 \rightarrow 3$) results in the prescribed instantaneous values of the phase currents (i_a^*, i_b^*, i_c^*).

$$\frac{di_d}{dt} = \frac{u_d}{L_s} - \frac{R_s}{L_s} i_d + p\omega_r i_q \quad (32)$$

$$\frac{di_q}{dt} = \frac{u_q}{L_s} - \frac{R_s}{L_s} i_q - p\omega_r i_d - \frac{p\omega_r \phi_0}{L_s} \quad (33)$$

This model describes the behavior of the synchronous motor in the system joint with the rotor (and the rotating field). To obtain the Matlab/Simulink model of the SPMSM that is similar to a real motor (powered in the fixed reference system), the model described by equations (32)–(33) must be preceded by two transformation blocks: ($3 \rightarrow 2$) $\text{si}(e^{-j})$, which transform the three-phase supply currents into a two-phase system (first transformation), rotating integrally with the rotor (second transformation). Following the integration of the equations of state in the rotating system, the phase currents are obtained by applying the inverse transformations (e^{+j}) and ($2 \rightarrow 3$). The model thus obtained is masked in the "SPMSM" block, in the general control scheme Fig.1. The estimation of the stator flux and rotor speed is performed in the "Gopinath Observer" block, a block that receives the two-phase stator currents of the motor as input. This rotor speed value is used both in the coordinate transformation blocks (for calculating the electrical angle θ_{r_estim}) and in the speed loop, thus closing the control chain.

4 Computation results

To validate the proposed control model, MATLAB/Simulink was used as a simulation environment, where several simulations were performed. The model also uses a SPMSM with the following constructive parameters, Table 1.

For the accuracy of the simulation results and their correct visualization on the graphs, 2s simulation times and a sampling step of 10ms were used. Thus, from the graph of rotor speed variation Fig.2 and electromagnetic torque, Fig.3, we can see the stability of the motor's operation with variations in stator resistance. Three values ($R_s = 0.5\Omega$; $R_s = 1\Omega$; $R_s = 1.5\Omega$) were imposed for this resistance and the operating dynamics of the motor were monitored over a 2s period.

Table 1. The parameters of SPMSM

$m=3$	$\phi_0 = 0.13Wb$
$R_s=1\Omega$	$J=0.01\text{ kgm}^2$
$L_s=0.0032\text{ H}$	$p=3$

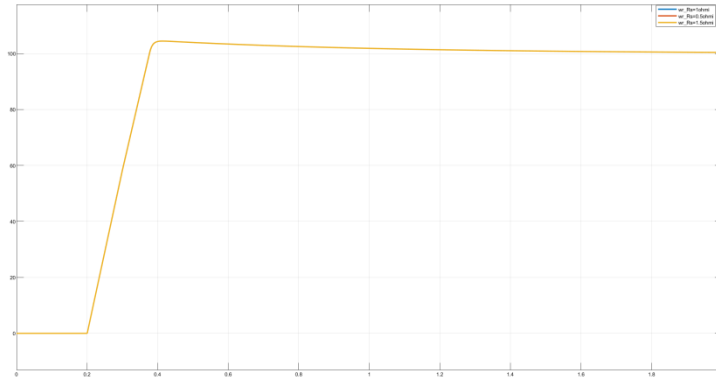


Figure 2. Rotor speed response for ($R_s = 0.5\Omega$; $R_s = 1\Omega$; $R_s = 1.5\Omega$)

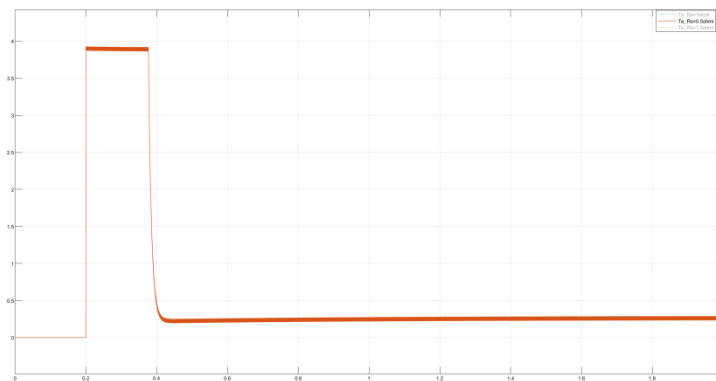


Figure 3. Electromagnetic torque response for ($R_s = 0.5\Omega$; $R_s = 1\Omega$; $R_s = 1.5\Omega$)

In the typical "current-controlled inverter" configuration we have an internal current regulation loop (PI/PI d-q decoupling), with high bandwidth compared to mechanical dynamics. The current regulator forces the d-q axis components of the current to follow the references, compensating for the terms in the electrical equations (dependent on R_s). Thus, for an ideal PI controller that is fast enough, the required control voltage is calculated by the controller so that the current error is zero: that is, the voltage required to compensate for the resistance losses and produce the desired current. So, the variation of R_s produces a change in the voltage value, but the regulator adjusts it to keep the electric current constant.

As observed from the specialized literature [14], in voltage control, the current is not fixed by a regulator — the stator resistance R_s enters directly into the equations of state and influences the current, and therefore the torque and speed.

Another problem analyzed in this paper is the confirmation of the robustness of the Gopinath-type flux observer implemented for the surface permanent magnet synchronous motor (SPMSM). The purpose of this analysis is to demonstrate the observer's ability to correctly estimate the stator flux components even under conditions of variations in machine parameters or rotation speed. Fig.4 illustrates the behavior of the system for a reference rotation speed $\omega_{ref} = 100 \text{ rad/sec}$ and $\omega_{ref} = 20 \text{ rad/sec}$, a regime in which the effects of nonlinearities and resistive terms become more pronounced, and flux estimation is normally more difficult. The obtained results show that the Gopinath observer provides a stable and precise estimation of the stator flux, without oscillations or drifts over time, which confirms the robustness of the method to speed variations and the influence of stator resistance.

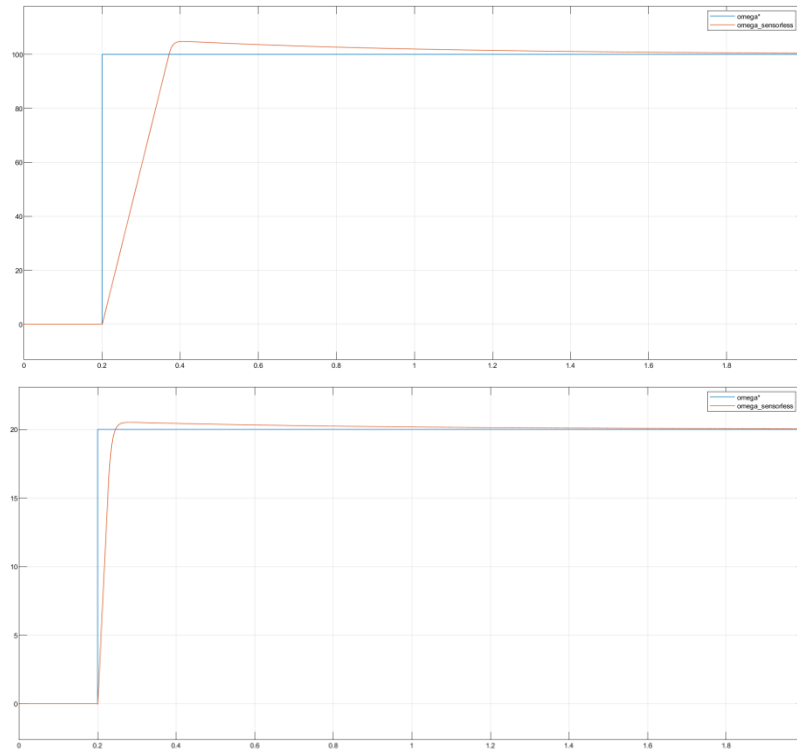


Figure 4. Speed dynamic response ($\omega_{ref} = 100\text{rad/sec}$, $\omega_{ref} = 20\text{rad/sec}$)

5 Conclusions

The paper presents an analysis of the performance of the vector control system applied to the permanent magnet synchronous motor (PMSM), with a focus on current control and the evaluation of the robustness of the Gopinath-type flux observer. The main objective is to investigate the stability of the system and the accuracy of rotor speed regulation in the presence of variations in electrical parameters, especially stator resistance.

The current control system uses separate loops to regulate the stator currents on the two orthogonal axes and to regulate the rotor speed. The performed simulations demonstrate that this structure ensures stability and high performance, maintaining the rotor speed independent of stator resistance variations. This confirms the efficiency and robustness of current control, providing a fast and precise response even to load changes or variable operating conditions.

The Gopinath flux observer, used for estimating stator flux and rotor speed, was evaluated in low-speed regimes ($\omega_{ref} = 20\text{rad/sec}$). The results indicate an accurate and stable estimate, demonstrating that the observer is robust to parametric variations and maintains consistent performance under practical operating conditions.

In conclusion, the study confirms that the combination of current vector control and the Gopinath observer provide a robust, stable and precise SPMSM drive system, capable of operating efficiently over the entire speed and load range. The obtained results can serve as a basis for practical implementations and for future research on adaptive observers and methods for online compensation of parametric variations.

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