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A Systematic Method of Generating Intuitive Bivariate Colour Legends

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Abstract. Colour is used increasingly often as quantitative indicator of combined continuous features such as temperature and humidity on geographical maps, or deformity on two axes in medical diagnosis. As such, generating flexible bivariate legends with particularly tough colour constraints for specific combinations of the variables has come to the forefront. The paper presents a systematic method of generating bivariate legends by first selecting a convenient colour subspace and then adjusting it through image processing techniques. The proposed method can generate a wide variety of legends with smart choices of parameters but remains simple enough to be worthy of consideration by specialists (in various fields) whose representations may benefit by becoming better intelligible. These legends can be used for pseudo-colouring of maps, diagrams, and many other images and graphical representations. An example is also provided for a comprehensive colouring to be used in the medical diagnosis of spine deformity.

Keywords: bivariate maps, pseudo-colouring, interpolation, warping

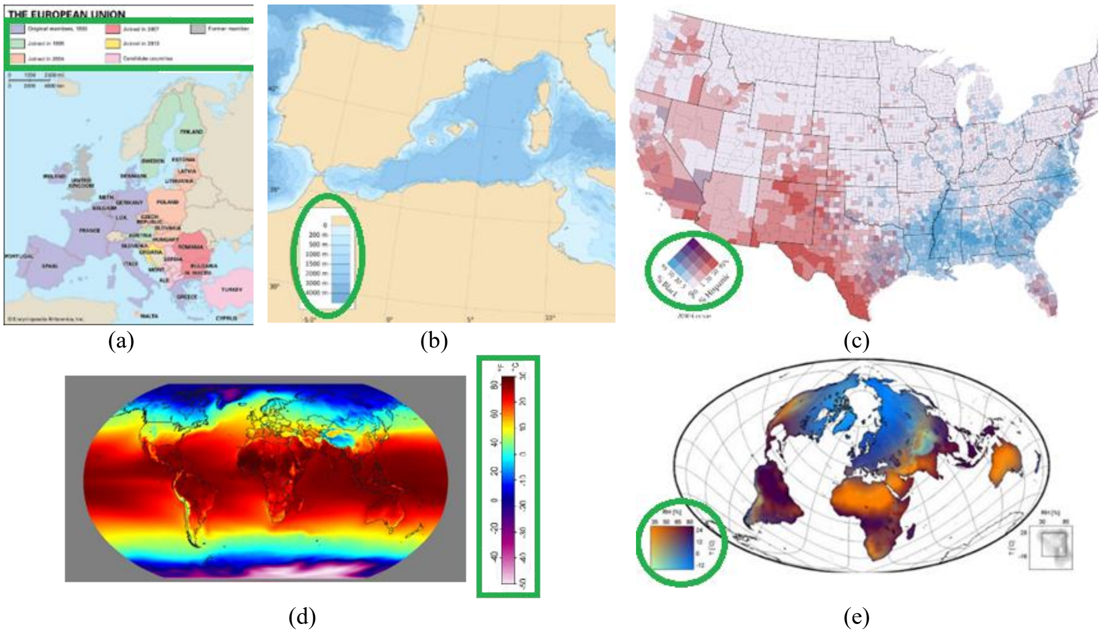
1. Introduction

Colour has had an important role in human evolution and can still be extremely influential in defining specific anthropocentric environments, summarizing information in memorable ways or inspiring associations between abstract concepts. Colour has been defined as a property of objects long before it was well understood [1]. The first known theory of colour belongs to Aristotle, who believed that light originated in the eye, that all colours came from lightness and darkness and that they were related to the four elements. Ptolemy also briefly commented on colours in his book on geometrical optics [2]. During the Islamic Golden Age, scholars like al-Kindi, al-Haytham, ibn-Sina and al-Tusi challenged Aristotle's view on the source of colour by emphasizing the role of external light [2][3]. Starting from the Early Renaissance, European painters and craftsman described a more practical theory based on paints, which eventually lead to the adoption of Red-Yellow-Blue (RYB) as the primary colours, still relevant in arts [4]. Although recognizing light as an external phenomenon, Galileo was inclined to place the existence of colour not outside, but rather within the mind of the observer [5]. However, with the prism experiments on splitting and recombining white light from Newton's Opticks and the thorough description of coloured light as a sliver of the electromagnetic spectrum, colour was firmly in the physics realm, described in terms of hue/frequency, saturation and brightness [6]. Yet, some colours such as *magenta* were not to be found in the rainbow spectrum either as single waves or as clusters of waves with similar frequencies. It was only after a more detailed analysis of the human (and animal) vision during the 19th and 20th centuries that the observer's importance was recognized once again, which in turn lead to the development of the colour system having primary colours Red-Green-Blue (RGB). Beyond that, artificial vision systems or data collecting devices (including satellites and space probes) are not limited to the visible spectrum, nor to the constraint of having only 3 (relatively wide) frequency bands.

Back in the 18th century, Goethe contested Newton's approach, arguing that colour is more than just a scientific measurement [7]. It is as much a physical property of light and human perception, as it is a subjective experience interpreted within a psychological frame. His view is still widely adopted by artists and producers of visual media. Colour symbolism, defined as subjective meanings of colours and the abstract concepts attached to them, varies between cultures, historic times and contexts. Therefore, the choice of colour for marketing purposes or for representing data in maps, interfaces and presentations might be critical for ensuring a good communication and retention of the contents, as well as improving analysis and usage by providing an intuitive frame of colour references [8]. In images and graphs, colour is used as a marker or emphasizer in three broad ways [9]:

- to separate discrete categories, such as assigning distinct colours to neighbouring countries on political maps or differentiation the members of the European Union by the joining year; for class discrimination, the colours just need to assure a good contrast and are otherwise free choices to convey additional meanings. Because there is no direct correlation between the colours of the classes, which are presented as a list of disjoint elements, the colour legend has no dimensionality.
- to represent sequential values of a feature in quantized levels, such as in simpler physical maps of the oceans where deeper shades of blue suggest larger bathymetric depths but using only a small number of shades overall (Fig. 1.b) or the percentage of populations (Fig. 1.c). Compared to the previous category, sequential colouring has a further constraint that the sequence of shades needs to be intuitively ordered (in both directions for 2D legends). Images containing this sort of legend on a geographic map fall under the common name of choropleth maps. Various applications have been developed in recent years to assist with building custom choropleth legends [10-14].
- to quantitatively represent the strength or intensity without any noticeable quantization, such as maps of temperature (as an example of 1D legend) or temperature-humidity (as an example of 2D legend). The constraint is that the colour gradient needs to be smooth, the colour legend becoming itself a colour map.

Quantitative 1D colour legends (palettes) are relatively easy to create from the natural order of hues in the spectrum, by creating lighter/darker shades of a primary colour or by concatenating two such ranges of shades. When more than one variable needs to be conveyed intuitively, the legend becomes bidimensional. Generating smooth gradients in every direction with constraints on specific points is useful when mapping sets of two independent continuous variables is useful in many fields, not just for geographical data [15].



(a) (b) (c)

(d) (e)

Figure 1. Examples of representation of geographical data with legends from each category
(a) European Union joining year (©Encyclopaedia Britanica)
(b) Western Mediterranean Sea bathymetry map [27]
(c) Distribution of Black and Hispanic populations in continental US [28]
(d) Annual average temperature ([29])
(e) Distribution of climate zones based on average temperature and relative humidity [20]

As the intent is to visualize the combined output, each variable combination must have a distinct colour, while at the same time very specific combinations of the variables must have a predetermined colour.

Fig. 1 shows examples of representation of geographical data with legends from each category. Please note that the focus is on the legend, marked by encirclement, not on the accuracy of data presented in the maps.

Attempts to build bivariate colour legends have been successfully proposed in the past, but had shortcomings such as having explicit granularity [16][17], limited flexibility [18] or limited applicability [19]. These methods are still useful for applications with a very small number of legends required and with softer constraints on the colours.

A custom bivariate colour map can be created even without a specialized application. Indeed, most historical legends have been created in this way, strictly in relation to the data represented, be they about geographical variables on maps [20], bioinformatics [21], medical diagnosis [22], engineering [23] or other fields.

In this paper, a more general framework is presented. The proposed method of generating bivariate colour legends is flexible enough to allow the generation of the legends described in the mentioned bibliography with smart choices of parameters, but still simple enough to be worthy of consideration by specialists (in other fields) whose representations may benefit from such legends.

The remainder of this paper is organized as follows: *Section 2* presents the proposed method, emphasizing the possible parameter choices. *Section 3* briefly discusses one-dimensional colour palettes within the framework of the method. *Section 4* exemplifies the generation of bivariate colour legends by describing an intuitive representation of spine diagnosis. The concluding remarks on the flexibility of the method and perspectives are explored in *Section 5*.

2. Proposed method

The proposed method is comprised of four steps, as shown in Fig. 2:

- selection of the colour space
- selection of constraint points
- interpolation
- warping

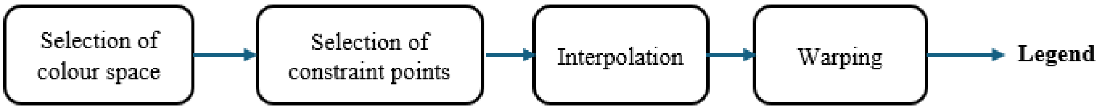
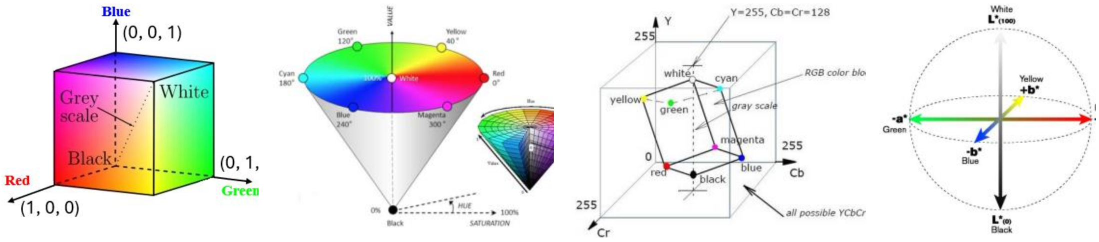


Figure 2. Diagram of the proposed method

2.1. Selection of the colour space

Generating colour legends with the explicit purpose of being used in graphic representations (meant to be visualised) restricts the colour space to finite three-dimensional linear spaces [24] such as RGB, HSV, YCbCr, CIE La*b* etc., in which every colour can be represented by a three-valued vector. Fig. 3 shows the mentioned colour spaces side by side. The spaces can be considered continuous from the perception of the human eye, although in practice they are used in digital environments more as discrete spaces with a very fine quantization.



RGB color space | HSV color space | YCbCr color space | CIE La*b* color space

Figure 3. Colour spaces [26]

The sum of two vectors from these spaces is either another colour, or outside the boundaries of the colours. In RGB, for instance, adding *green* to *red* generates *yellow* ($[1,0,0]^T + [0,1,0]^T = [1,1,0]^T$), but adding *magenta* to *red* is outside the boundaries of the colour cube. Most of the examples in the next section will use RGB as the space of choice, out of which the bivariate legends will be extracted.

2.2. Selection of constraint points

The selection of constraint points is dependent on the contents presented and the desired colour symbolism. For physical features in maps, it is customary to have shades of *green* for planes, *brown* for mountains and *blue* for the seas. For temperature, *red* and *orange* are usually considered hot, while *blue* is considered cold. For the general assessment of situations, the semaphore conventions indicate *red* as dangerous or imperative, *yellow* as worrisome or cautionary, *green* as fine or harmonious and *blue* as excellent or exceptionally well. For internal states, *red* is associated with passion or anger, *yellow* with either optimism or energy and so on.

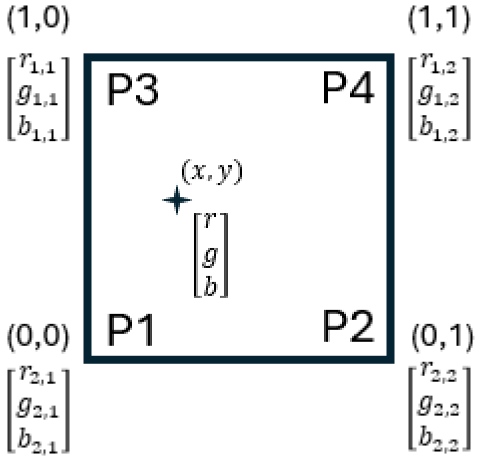


Figure 4. Representation of the legend in terms of geometrical coordinates and colour vectors

Table 1. Correspondence between description, geometrical coordinates and colour vectors

Corner position	**Coordinates (x, y)**	**Colour (r, g, b)**
Lower left (P1)	(0,0)	$\begin{bmatrix} r_{2,1} \\ g_{2,1} \\ b_{2,1} \end{bmatrix}$
Lower right (P2)	(0,1)	$\begin{bmatrix} r_{2,2} \\ g_{2,2} \\ b_{2,2} \end{bmatrix}$
Upper left (P3)	(1,0)	$\begin{bmatrix} r_{1,1} \\ g_{1,1} \\ b_{1,1} \end{bmatrix}$
Upper right (P4)	(1,1)	$\begin{bmatrix} r_{1,2} \\ g_{1,2} \\ b_{1,2} \end{bmatrix}$

The number of constraint points can also vary. For univariate legends, two or three constraints (the edges and maybe the middle colour) are usually enough. For bivariate legends, the simplest maps use only four corner points, but the number can also be increased for more specific colours at various positions. Without loss of generality, we can assume that 2D representation of the legend is a square with side 1 with geometric coordinates and colours indicated in Fig. 4 and Table 1, and an interior point at coordinates (x, y) will be coloured $[r,\ g,\ b]^T$.

2.3. Interpolation

Probably the most intuitive computation of inner point for all legends is linear/bilinear interpolation, which will be used in most examples from the next sections. Under bilinear interpolation, four coplanar corner constraints lead to legends contained into slices of the colour space, while four non-coplanar corner constraints lead to shapes similar to a twisted sheet.

2.3.1. Four (corner) constraints

The computations for determining the colour at each position are defined by equations (1-3). An interior point of the legend, of geometric coordinates (x, y), will have the colour $[r, g, b]^{\mathrm{T}}$ as computed.

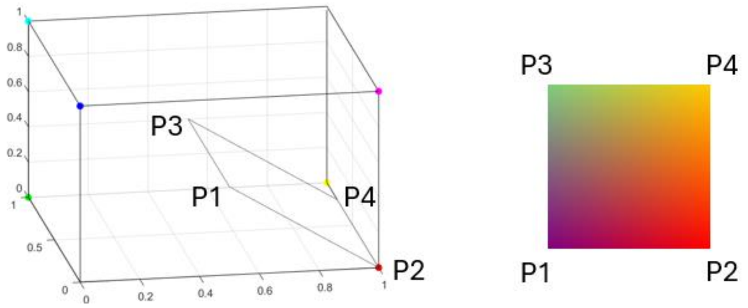
$$r = \begin{bmatrix} 1-x & x \end{bmatrix} \begin{bmatrix} r_{1,1} & r_{1,2} \\ r_{2,1} & r_{2,2} \end{bmatrix} \begin{bmatrix} 1-y \\ y \end{bmatrix} \tag{1}$$

$$g = \begin{bmatrix} 1-x & x \end{bmatrix} \begin{bmatrix} g_{1,1} & g_{1,2} \\ g_{2,1} & g_{2,2} \end{bmatrix} \begin{bmatrix} 1-y \\ y \end{bmatrix} \tag{2}$$

$$b = \begin{bmatrix} 1-x & x \end{bmatrix} \begin{bmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{bmatrix} \begin{bmatrix} 1-y \\ y \end{bmatrix} \tag{3}$$

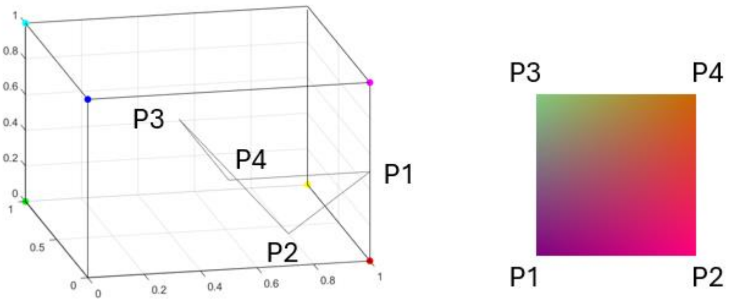
Fig. 5 and Fig. 6 show examples of legends generated from coplanar corners and non-coplanar corners, respectively. The RGB space is used for convenience, but the equations would work for other three-dimensional spaces as well.

The condition of having distinct colour for each point in the legend is achieved when opposing sides do not intersect in the 3D representation.



(a) (b)

Figure 5. Bivariate legend generated from coplanar corner constraints:
(a) view inside the RGB cube, (b) view as legend

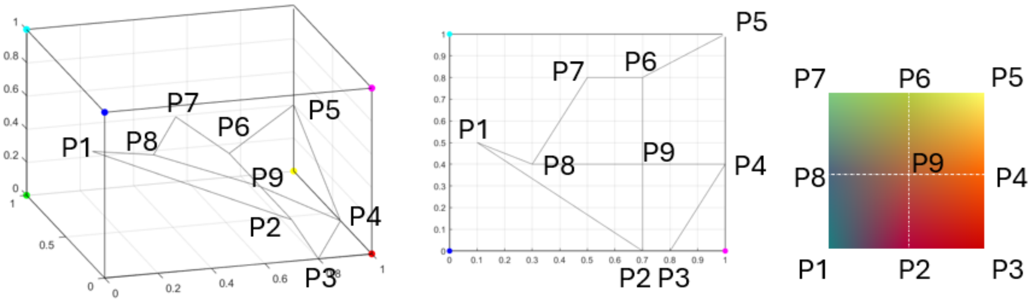


(a) (b)

Figure 6. Bivariate legend generated from non-coplanar corner constraints:
(a) view inside the RGB cube, (b) view as legend

2.3.2. Multiple constraints

Multiple constraints can generate piecewise-interpolated legends by using subsets of four constraints (patches), as shown in Fig. 7. If the patches of the legend are not supposed to be rectangular, the next step (warping) would provide a way of distorting the legend as needed.



(a) (b) (c)

Figure 7. Bivariate legend generated from sets of non-coplanar corner constraints (patches):
(a) view inside the RGB cube, (b) top view of the RGB cube, (c) view as legend

There are also other types of interpolation, such as polynomial or spline, which may also create interesting legends under the condition that the map does not intersect or fold onto itself, so that each colour in the legend is unique.

2.4. Warping

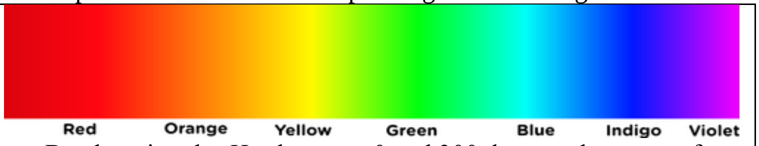
The final step considers the legends as images. Warping in this case refers to the any image processing technique that maps the image onto itself bijectively, preserving point connections but not necessarily Euclidean distances [25]. Any distortions, including stretching, triangle meshing and grid warping, are allowed as long as folding does not occur. If the legend is considered satisfactory after the initial interpolation step, or if the significance of the intermediate values mapped by the legend can be adjusted naturally outside the legend, then warping can even be skipped.

Interpolation and warping could theoretically be combined into a single step, but it is more intuitive to separate the selection of colours (that make up the legend and can be visualized inside the 3D colour space) from the processing of the legend (with the purpose of emphasizing regions of colour).

3. Univariate colour legends

Quantitative 1D colour palettes are relatively easy to create from the natural order of hues in the spectrum or by creating lighter/darker shades of a primary colour. *Table 2* shows several examples of 1D colour legends.

Table 2. Examples of sets of parameters and the corresponding univariate legends

Rainbow palette: Colour space: HSV Endpoints: $[0, 1, 0.5]^T, [300, 1, 0.5]^T$ Interpolation: linear Warping: none	 By changing the *Hue* between 0 and 300 degrees the range of colours spans the entire rainbow
Grayscale palette: Colour space: RGB Endpoints: $[0, 0, 0]^T, [1, 1, 1]^T$ Interpolation: linear Warping: none	
Blue shades palette: Colour space: RGB Endpoints: $[0, 0, 1]^T, [1, 1, 1]^T$ Interpolation: linear Warping: none	
Red to white to blue palette: Colour space: RGB Endpoints patch 1: $[0.5, 0, 0]^T, [1, 1, 1]^T$ Endpoints patch 2: $[1, 1, 1]^T, [0, 0, 1]^T$ Interpolation: linear Warping: stretching (blue)	

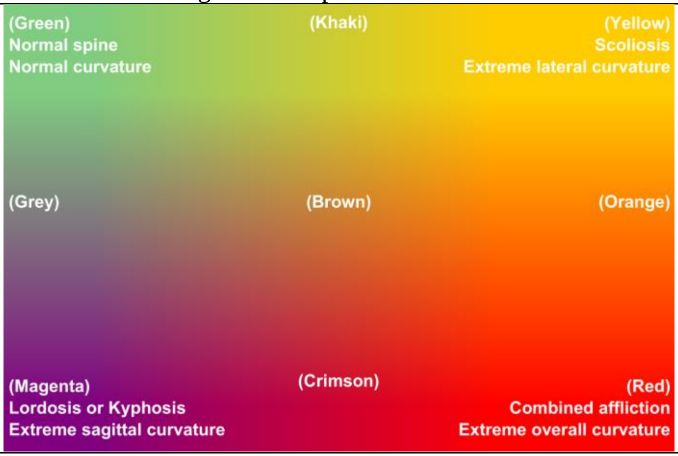
4. Case study: *Intuitive bivariate colour legend in spinal disorders*

As an example, we have situation in the medical field where two spine afflictions can indicate (relatively) independent problems [22]. The intensities of the two types of afflictions along the spine, *scoliosis* and *kyphosis*, are represented with colour gradients *green-yellow* (for scoliosis in the frontal view) and *green-magenta* (for kyphosis in the sagittal view), but their colours can be combined with the help of the bivariate legend shown in *Table 3*. On top of the colours, the legend also presents possible interpretations of the colours, depending on the images where they are used. As shown in Fig. 8.b, the colours can be used to indicate the severity of the affliction. Fig. 8.c uses the colours to indicate normal or extreme curvature on regions of the spine, either in the frontal plane Fig. 8.c (left) or the sagittal plane Fig. 8.c (right), then mixing them according to the legend for the combined stress on the spine in Fig. 8.c (middle).

The map considers the semaphore conventions, having a shade of *green* (no affliction) as the good corner and *yellow*, *magenta*, and *red* as the remaining corners, indicating extreme afflictions.

The four corners in this case are co-planar and not too dark.

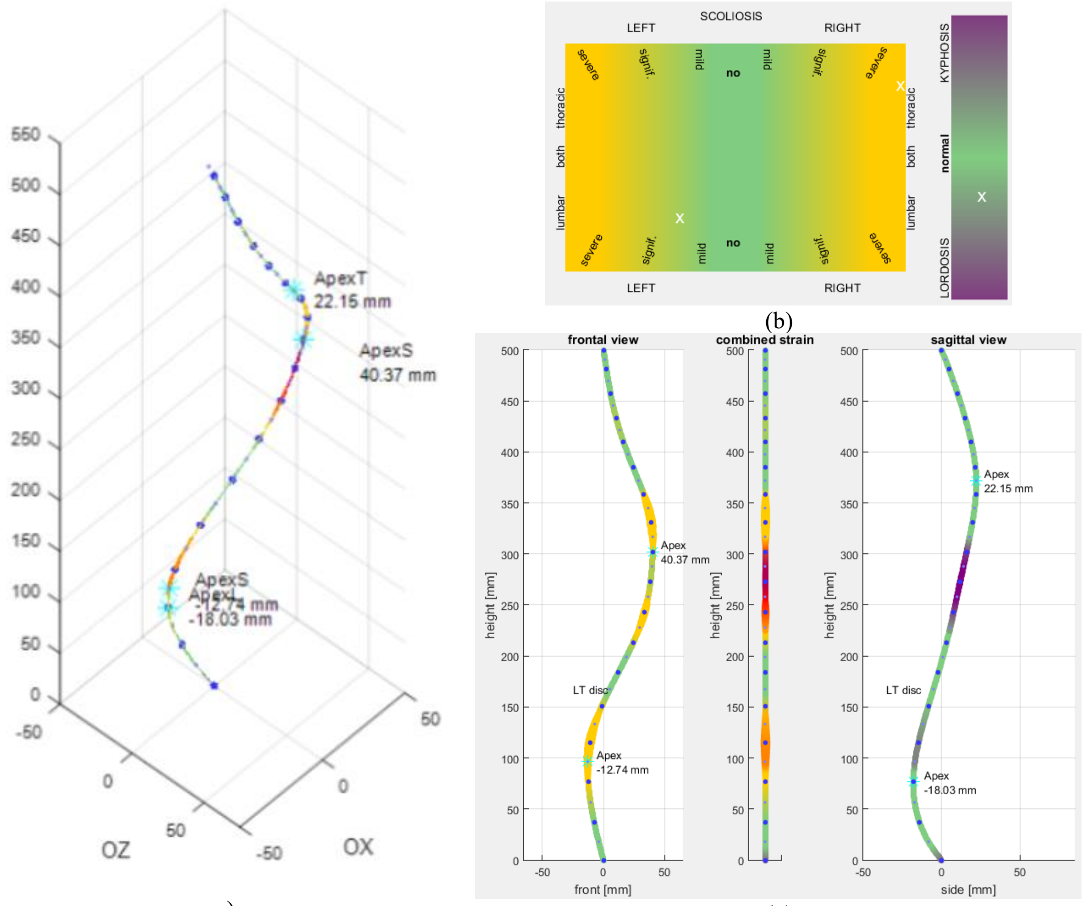
Table 3. Examples of a set of parameters and the corresponding bivariate legend with medical diagnosis interpretations

Colour space: **RGB** *Top Corners:* Top left: $[0.5, 0.8, 0.5]^T$ Top right: $[1, 0.8, 0]^T$ *Bottom corners:* Bottom left: $[0.5, 0, 0.5]^T$ Bottom right: $[1, 0, 0]^T$ Interpolation: *linear* Warping: *none*	

Generating the map with linear interpolation creates colour gradients that are compatible with the meaning of the position:

- a lack of afflictions is marked with shades of *green*
- medium individual afflictions are neutral colours *grey* and *khaki*
- combined medium afflictions are *murky browns*
- severe individual afflictions are *yellow* and *magenta*
- extreme combined afflictions are increasingly more *orange* and *crimson* until the *bright red* indicating extreme two-variate afflictions

In terms of computational description, this legend is especially intuitive because the horizontal affliction changes only the red and blue layers, whereas the vertical affliction changes the green layer.



a)

(b)

(c)

Figure 8. (a) 3D representation of the spine (pseudo-coloured),
(b) Scoliosis and Kyphosis diagnostic with separate colour codes
(c) frontal and sagittal view with separate colour codes, combined strain with combined colour code
(using the legend from *Table 3*)

5. Conclusions

The presented method for generating bivariate legends is both reliable and flexible. As shown in previous section, smart choices of parameters enable the generation of other maps discussed in the introduction, as well as providing an option to further adapt the legends to more specific needs through warping.

The method can be further generalized by considering other colour spaces (although RGB and HSV are still the most common for visualization purposes), other interpolation techniques or new parametric / non-parametric warping distortions.

Although more difficult to visualize, tri-variate legends can be constructed as subspaces of the three-valued colour spaces (the most intuitive being the RGB cube itself), by defining the eight corners of the map. However, such legends can be difficult to represent in 2D and it might be simpler to explain the axes of the legend in words. Multivariate legends are mathematically extractable from multi-valued (pseudo-)colour spaces, but in order to have the information intelligible to users it may be useful in many cases to extract meaning though data processing techniques (such as *Principal Component Analysis*) and then revert to bivariate legends, if possible.

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